# ANSWERS

## 1. DIFFERENTIATION

### EXERCISE 1.1

(1) (i) $5\,(3x^2-2)\,(x^3-2x-1)^4$

(ii) $\frac{5}{2}\left(3\sqrt{x}-4\sqrt[3]{x}\right)\left(2x^{\frac{3}{2}}-3x^{\frac{4}{3}}-5\right)^{\frac{3}{2}}$

(iii) $\frac{x+2}{\sqrt{x^2+4x-7}}$

(iv) $\frac{x\,(2\sqrt{x^2+1}+1)}{2\sqrt{x^2+1}\cdot\sqrt{x^2+\sqrt{x^2+1}}}$

(v) $-\frac{4x-7}{(2x^2-7x-5)^{\frac{8}{3}}}$

(vi) $\frac{15\,(3x-4)}{2\,(3x-5)^{\frac{3}{2}}}\left(\sqrt{3x-5}-\frac{1}{\sqrt{3x-5}}\right)^4$

(2) (i) $-2x\sin\left(x^2+a^2\right)$ (ii) $\frac{3e^{(3x+2)}}{2\sqrt{e^{(3x+2)}+5}}$

(iii) $\operatorname{cosec} x$ (iv) $\frac{\sec^2\sqrt{x}}{4\sqrt{x}\cdot\sqrt{\tan\sqrt{x}}}$

(v) $\frac{-9\operatorname{cosec}^2[\log(x^3)]\cdot\cot^2[\log(x^3)]}{x}$

(vi) $3\sin^2 x\cdot\cos x\cdot 5^{\sin^3 x+3}\cdot\log 5$

(vii) $\frac{\sin x\operatorname{cosec}\sqrt{\cos x}\cdot\cot(\sqrt{\cos x})}{2\sqrt{\cos x}}$

(viii) $-3x^2\tan(x^3-5)$

(ix) $5\sin 2x\cdot e^{3\sin^2 x-2\cos^2 x}$

(x) $\frac{-2x\cdot\sin[2\log(x^2+7)]}{x^2+7}$

(xi) $-\sec^2\left[\cos(\sin x)\right]\cdot\sin(\sin x)\cdot\cos x$

(xii) $4x^3\cdot\sec^2(x^4+4)\cdot\sec\left[\tan(x^4+4)\right]\cdot\tan\left[\tan(x^4+4)\right]$

(xiii) $\frac{2\log x}{x}-\frac{2}{x}$

(xiv) $\frac{\cos\sqrt{\sin\sqrt{x}}\cdot\cos\sqrt{x}}{4\sqrt{x}\cdot\sqrt{\sin\sqrt{x}}}$

(xv) $2x\cdot e^{x^2}\left[\tan\left(e^{x^2}\right)\right]$ (xvi) $\frac{1}{2x\log x}$

(xvii) $\frac{2\left[\log[\log(\log x)]\right]}{x\log x\cdot\log(\log x)}$

(xviii) $4x\sin(2x^2)$

(3) (i) $6\,(x+2)(x^2+4x+1)^2+4\,(3x^2-5)\,(x^3-5x-2)^3$

(ii) $8\,(1-2x)\,(1+4x)^5\,(3+x-x^2)^7+20\,(1+4x)^4\,(3+x-x^2)^8$

(iii) $\frac{14-3x}{2(7-3x)^{\frac{3}{2}}}$ (iv) $\frac{6x^2\,(x^3+15)\,(x^3-5)^4}{(x^3+3)^6}$

(v) $\sin 2x\,(1+\sin^2 x)\,(1+\cos^2 x)^2\,(1-5\sin^2 x)$

(vi) $-\frac{\sin x}{2\sqrt{\cos x}}-\frac{\sin\sqrt{x}}{4\sqrt{x}\cdot\sqrt{\cos\sqrt{x}}}$

(vii) $3\sec 3x$ (viii) $\frac{\pi\cos x^\circ}{90\,(1-\sin x^\circ)^2}$

(ix) $-\frac{\operatorname{cosec}^2\left(\frac{\log x}{2}\right)}{2x}+\tan x\cdot\operatorname{cosec}^2 x$

(x) $\frac{8e^{4x}}{(e^{4x}+1)^2}$ (xi) $-\frac{e^{\sqrt{x}}}{\sqrt{x}\,(e^{\sqrt{x}}-1)^2}$

(xii) $6\operatorname{cosec} 2x+4\cot x+\frac{14x}{x^2+7}$

(xiii) $3\operatorname{cosec} 3x$ (xiv) $-\frac{5}{2}\operatorname{cosec}\left(\frac{5x}{2}\right)$

(xv) $-\sec x$

(xvi) $2\log 4 + \dfrac{3x}{x^2+5} - \dfrac{9x^2}{2(2x^3-4)}$

(xvii) $2x - \dfrac{6}{5-4x} + \dfrac{2}{7-6x}$

(xviii) $-\sin x \log a - \dfrac{6x}{x^2-3} - \dfrac{1}{x \log x}$

(xix) 0 (xx) $\dfrac{x(x^2+2)^3(7x^2+38)}{(x^2+5)^{\frac{3}{2}}}$

(4) (i) $-16$ (ii) 35 (iii) $-20$ (iv) 28

(5) $-5$ (6) $\dfrac{12}{5}$ (7) $x = 0$ or $\dfrac{2\pi}{3}$ or $2\pi$

(8) $e^{2x} + 6e^x + 14$, $e^{x^2+5}$, $2x$, $e^x$, $f'[g(x)]\cdot g'(x)$, $2e^{2x} + 6e^x$, 8, $g'[f(x)]\cdot f'(x)$, $2xe^{x^2+5}$, $-2e^6$.

## EXERCISE 1.2

(1) (i) $\dfrac{1}{2\sqrt{x}}$ (ii) $-\dfrac{1}{4\sqrt{x}\sqrt{2-\sqrt{x}}}$

(iii) $\dfrac{1}{3\sqrt[3]{(x-2)^2}}$, for $x > 2$ (iv) $\dfrac{2}{2x-1}$

(v) 2 (vi) $e^x$

(vii) $2e^{2x-3}$ (viii) $\dfrac{1}{x \log 2}$

(2) (i) $\dfrac{1}{x \cdot e^x (x+2)}$ (ii) $\dfrac{1}{\cos x - x \sin x}$

(iii) $\dfrac{1}{7^x(x \log 7 + 1)}$ (iv) $\dfrac{x}{2x^2+1}$

(v) $\dfrac{1}{1+\log x}$

(3) (i) $\dfrac{1}{14}$ (ii) $\dfrac{1}{4}$ (iii) $\dfrac{1}{12}$ (iv) $\dfrac{1}{5}$

(4) 1

(5) (i) and (ii) derivative proved.

(6) (i) $\dfrac{1}{x[1+(\log x)^2]}$ (ii) $\dfrac{e^x}{\sqrt{1-e^{2x}}}$

(iii) $-\dfrac{3x^2}{1+x^6}$ (iv) $-\dfrac{4^x \log 4}{1+4^{2x}}$

(v) $\dfrac{1}{2\sqrt{x}(1+x)}$ (vi) $\dfrac{x}{\sqrt{1-x^4}}$

(vii) $\dfrac{2}{\sqrt{2-x^2}}$ (viii) $\dfrac{3\sqrt{x}}{2\sqrt{1-x^3}}$

(ix) $9x^8$ (x) $2x$

(7) (i) $2xe^{x^2}$ (ii) $-5^x \log 5$ (iii) $\dfrac{1}{2}$

(iv) $-x$ (v) $-\dfrac{1}{2}$ (vi) $-6$

(vii) $-\dfrac{1}{6}$ (viii) $-\dfrac{3}{2}$ (ix) $-\dfrac{7}{2}$

(x) $-\dfrac{1}{2}$ (xi) $-\dfrac{1}{2}$ (xii) $\dfrac{2}{3}$

(8) (i) 1 (ii) 1 (iii) $\dfrac{1}{2\sqrt{x}}$

(iv) 3 (v) $e^x$ (vi) $2^x \log 2$

(9) (i) $\dfrac{2}{1+x^2}$ (ii) $\dfrac{2}{1+x^2}$

(iii) $-\dfrac{2}{1+x^2}$ (iv) $\pm\dfrac{2}{\sqrt{1-x^2}}$

(v) $-\dfrac{3}{\sqrt{1-x^2}}$ (vi) $-\dfrac{2e^x}{1+e^{2x}}$

(vii) $\dfrac{2\cdot 3^x \log 3}{1+3^{2x}}$

(viii) $\dfrac{2\cdot 4^x \log 4}{1+4^{2x}}$ or $\left(\dfrac{4^{x+\frac{1}{2}} \log 4}{1+4^{2x}}\right)$

(ix) $-\dfrac{10}{1+25x^2}$ (x) $-\dfrac{3\sqrt{x}}{1+x^3}$

(xi) $\dfrac{5x\sqrt{x}}{1+x^5}$ (xii) $\dfrac{1}{2\sqrt{x}(1+x)}$

(10) (i) $\dfrac{3}{1+9x^2} + \dfrac{5}{1+25x^2}$

(ii) $\frac{7}{1+49x^2}-\frac{5}{1+25x^2}$

(iii) $\frac{1}{2\sqrt{x}}\left(\frac{3}{1+9x}-\frac{1}{1+x}\right)$

(iv) $2^x \log 2\left(\frac{3}{1+9(2^{2x})}+\frac{1}{1+2^{2x}}\right)$

(v) $2^x \log 2\left(\frac{2}{1+4(2^{2x})}-\frac{1}{1+2^{2x}}\right)$

(vi) $\frac{3a}{a^2+9x^2}+\frac{2a}{a^2+4x^2}$ (vii) 1

(viii) $\frac{2}{1+(2x+1)^2}-\frac{3}{1+(3x-4)^2}$

(ix) $\frac{2}{1+(2x+3)^2}+\frac{1}{1+(x-1)^2}$

## EXERCISE 1.3

(1) (i) $\frac{(x+1)^2}{(x+3)^3(x+3)^4}\left[\frac{2}{x+1}-\frac{3}{x+2}-\frac{4}{x+3}\right]$

(ii) $\frac{1}{3}\sqrt[3]{\frac{4x-1}{(2x+3)(5-2x)^2}}\left(\frac{4}{4x-1}-\frac{2}{2x+3}+\frac{4}{5-2x}\right)$

(iii) $(x^2+3)^{\frac{3}{2}}\cdot\sin^3 2x\cdot 2^{x^2}\left[\frac{3x}{x^2+3}+6\cot 2x+2x\log 2\right]$

(iv) $\frac{(x^2+2x+2)^{\frac{3}{2}}}{(\sqrt{x}+3)^3(\cos x)^x}\left[\frac{3(x+1)}{x^2+2x+2}-\frac{3}{2\sqrt{x}(\sqrt{x}+3)}+x\tan x-\log(\cos x)\right]$

(v) $\frac{x^5\cdot\tan^3 4x}{\sin^2 3x}\left[\frac{5}{x}+24\operatorname{cosec} 8x-6\cot 3x\right]$

(vi) $x^{\tan^{-1}x}\left[\frac{\tan^{-1}x}{x}+\frac{\log x}{1+x^2}\right]$

(vii) $\sin^x x \quad [x\cot x+\log(\sin x)]$

(viii) $\cos(x^x)\cdot x^x(1+\log x)$

(2) (i) $ex^{e-1}+e^x+x^x(1+\log x)$

(ii) $x^{x^x}\cdot x^x\cdot\log x\left[1+\log x+\frac{1}{x\log x}\right]+e^{x^x}\cdot x^x(1+\log x)$

(iii) $(\log x)^x\left[\frac{1}{\log x}+\log(\log x)\right]+(\cos x)^{\cot x}[1+\operatorname{cosec}^2 x\log(\cos x)]$

(iv) $x^{e^x}\cdot e^x\left[\frac{1}{x}+\log x\right]+(\log x)^{\sin x}\left[\frac{\sin x}{x\log x}+\cos x\log(\log x)\right]$

(v) $\sec^2 x\cdot e^{\tan x}+(\log x)^{\tan x}\left[\frac{\tan x}{x\log x}+\sec^2 x\log(\log x)\right]$

(vi) $(\sin x)^{\tan x}[1+\sec^2 x\log(\sin x)]+(\cos x)^{\cot x}[1+\operatorname{cosec}^2 x\log(\cos x)]$

(vii) $10^{x^x}x^x\log 10\,(1+\log x)+x^{x^{10}}\cdot x^9(1+10\log x)+x^{10^x}\cdot 10^x\left(\frac{1}{x}+\log x\cdot\log 10\right)$

(viii) 2

(3) (i) $-\sqrt{\frac{y}{x}}$ (ii) $-\sqrt{\frac{x}{y}}$

(iii) $-\frac{\sqrt{y}\,(2\sqrt{x}+\sqrt{y}\,)}{\sqrt{x}\,(2\sqrt{y}+\sqrt{x}\,)}$ (iv) $-\frac{3x^2+2xy+y^2}{x^2+2xy+3y^2}$

(v) $-\frac{y}{x}$ (vi) $-\frac{e^y+ye^x}{e^x+xe^y}$

(vii) $\frac{\sin(x-y)+e^{x+y}}{\sin(x-y)-e^{x+y}}$ (viii) $-\frac{1+y\sin(xy)}{1+x\sin(xy)}$

(ix) $\frac{y(1-xe^{x-y})}{x(1-ye^{x-y})}$

(x) $\frac{\sin(x-y)-\cos(x+y)-1}{\sin(x-y)+\cos(x+y)-1}$

## EXERCISE 1.4

(1) (i) $\frac{1}{t}$ (ii) $\frac{b}{a}\cos\theta$ (iii) $\frac{2}{\sqrt{a^2+m^2}}$

(iv) $\sec^3\theta$ (v) $\frac{b}{a}\tan\left(\frac{\theta}{2}\right)$

(vi) $\frac{y(t^2+1)\log a}{axt}$ (vii) $-\frac{1}{2}$ (viii) $\frac{1}{3}$

(2) (i) $\frac{3\sqrt{3}}{2}$ (ii) $-\sqrt{3}$ (iii) $-\frac{\pi}{6}$

(iv) $1-\sqrt{2}$ (v) $3+\pi$

(4) (i) $\frac{x\cos x+\sin x}{\sec^2 x}$ (ii) 1

(iii) $-\frac{1}{2}$ (iv) 2 (v) $-x(\log x)^2\cdot 3^x$

(vi) $-\frac{x\sqrt{x^2-1}}{2}$

(vii) $\frac{(1+\log x)\cdot x^{x+1-\sin x}}{\sin x+x\cos x\cdot\log x}$

(viii) $\frac{\sqrt{1-x^2}}{4(1+x^2)}$

## EXERCISE 1.5

(1) (i) $40x^3-24x-\frac{12}{x^4}$

(ii) $2e^{2x}(1+\tan x)\cdot(2+\tan x+\tan^2 x)$

(iii) $-e^{4x}(9\cos 5x+40\sin 5x)$

(iv) $x(5+6\log x)$ (v) $-\frac{1+\log x}{(x\log x)^2}$

(iv) $x^{x-1}+x^x(1+\log x)^2$

(2) (i) $-\frac{1}{4a}\operatorname{cosec}^4\left(\frac{\theta}{2}\right)$ (ii) $-\frac{1}{4at^3}$

(iii) 6 (iv) $-\frac{2\sqrt{2}\,b}{a^2}$

(4) (i) $\frac{d^ny}{dx^n}=\frac{m!\,a^n(ax+b)^{m-n}}{(m-n)!}$ if $m>0, m>n$,

$\frac{d^ny}{dx^n}=0$ if $m>0, m<n$

$\frac{d^ny}{dx^n}=n!\,a^n$ if $m>0, m=n$

(ii) $\frac{(-1)^n n!}{x^{n+1}}$ (iii) $a^ne^{ax+b}$

(iv) $p^na^{px+q}(\log a)^n$

(v) $\frac{(-1)^{n-1}(n-1)!\,a^n}{(ax+b)^n}$ (vi) $\cos\left(\frac{n\pi}{2}+x\right)$

(vii) $a^n\sin\left(\frac{n\pi}{2}+ax+b\right)$

(viii) $(-2)^n\cos\left(\frac{n\pi}{2}+3-2x\right)$

(ix) $\frac{(-1)^{n-1}(n-1)!\,2^n}{(2x+3)^n}$

(x) $\frac{(-1)^n\cdot n!\cdot 3^n}{(3x-5)^{n+1}}$

(xi) $e^{ax}(a^2+b^2)^{\frac{n}{2}}\cdot\cos\left[bx+c+n\tan^{-1}\left(\frac{b}{a}\right)\right]$

(xii) $e^{8x}\cdot(10)^n\cos\left[6x+7+n\tan^{-1}\left(\frac{3}{4}\right)\right]$

## MISCELLANEOUS EXERCISE 1

**(I)**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| D | C | C | B | A | C | D | C | B | C | A | B |

**(II)** (1) $\frac{3}{4}$ (ii) Does not exist (iii) $-2$

(iv) $-\frac{1}{2\cdot\sqrt{1-x^2}}$ [Hint : $x = \cos 2\theta$]

(v) $\frac{3}{1+9x^2}+\frac{5}{1+25x^2}$

(vi) $\frac{1}{2(1+x^2)}$ [Hint : $x = \tan\theta$]

(2) (A) 3, (B) 5, (C) 4, (D) 1.

(3) (i) $-\frac{1}{9}$ (ii) $-\frac{40}{3}$ (iii) $-\frac{29}{96}$

(iv) $-\frac{4}{9}$

(4) (i) $-\frac{x}{\sqrt{1-x^2}}$ [Hint : $x = \cos 2\theta$]

(ii) $-\frac{1}{2}$ [Hint : $x = \cos 2\theta$]

(iii) $\frac{3}{2\sqrt{x}(1+x)}$ [Hint : $\sqrt{x} = \tan\theta$]

(6) (i) $\frac{\sqrt{1-x^2}}{4(1+x^2)}$

(ii) $-\frac{2x}{\sqrt{1+x^2}.\sin(\log x)}$ (iii) 1

## 2. APPLICATIONS OF DERIVATIVES

### EXERCISE 2.1

(1) (i) $2x-y+4=0,\ x+2y-8=0$

(ii) $4x-5y+12=0,\ 5x+4y-26=0,$

(iii) $y=2,\ x=\sqrt{3}$

(iv) $\pi x+2y-2\pi=0,$

$4x-2\pi y+\pi^2-4=0$

(v) $2x-y=0,\ 4x+8y-5\pi=0$

(vi) $4x+2y-3=0,\ 2x-4y+1=0$

(vii) $17x-4y-20=0,\ 8x+34y-135=0$

(2) (4, 1)

(3) $(2,-2)\left(-\frac{2}{3},-\frac{14}{27}\right)$ (4) $y=0$ and $y=4$

(5) $x+3y-8=0,\ x+3y+8=0$

(6) $a=2,\ b=-7$ (7) (4, 11) and $\left(-4,-\frac{31}{3}\right)$

(8) $0.8\ \pi$ cm$^2$/sec. (9) 6 cm$^3$/ sec.

(10) $\frac{3\sqrt{6}}{2}$ cm$^2$/ sec. (11) 8 cm$^2$/ sec

(12) 7.2 cm$^3$/ sec (13) 3 km/hr

(14) (i) $\left(\frac{3}{8}\right)$ meter/sec. (ii) $\frac{9}{8}$ meter/sec.

(15) 0.9 meter/sec. (16) $\left(\frac{4\pi}{3}\right)$ cm$^3$/ sec

## EXERCISE 2.2

(1) (i) 2.9168 (ii) 3.03704 (iii) 1.9997
(iv) 248.32 (v) 64.48

(2) (i) 0.953 (ii) 0.42423 (iii) 0.4924
(iv) 1.02334

(3) (i) 0.7845 (ii) 0.7859 (iii) 0.7859

(4) (i) 2.70471 (ii) 8.1279 (iii) 9.09887

(5) (i) 4.6152 (ii) 2.1983 (iii) 3.006049

(6) (i) 6.91 (ii) 9.72

## EXERCISE 2.3

(1) (i) Valid (ii) Valid
(iii) Invalid (iv) Valid
(v) Invalid (vi) Invalid

(2) $b = 1$

(3) (i) $\frac{\pi}{4}$ or $\frac{5\pi}{4}$ (ii) $c = \pi$ (iii) $c = \frac{5}{2}$

(4) $p = -6, q = 11$ (6) $c = -2$

(7) (i) $e - 1$ (ii) $2 \pm \frac{2}{\sqrt{3}}$ (iii) $\frac{1}{7}$
(iv) $\frac{1}{2}$ (v) $3 + \sqrt{2}$

## EXERCISE 2.4

(1) (i) Increasing $\forall\ x \in R$
(ii) Decreasing $\forall\ x \in R$
(iii) Increasing $\forall\ x \in R$

(2) (i) $x < -1$ and $x > 2$ (ii) $R - \{1\}$
(iii) $x < -2$ and $x > 6$

(3) (i) $-1 < x < 2$ (ii) $(-5, 5) - \{0\}$
(iii) $x \in (2, 4)$

(4) (a) $(-\infty, -4] \cup [12, \infty)$
(b) $-4 \leq x \leq 12$ i.e. $[-4, 12]$

(5) (a) $x < -3$ and $x > 8$ (b) $-3 < x < 8$

(6) (a) $-1 < x < 1$ (b) $(-\infty, -1) \cup (1, \infty)$

(9) (i) Max $= \frac{36}{25}$, Min $= -\frac{16}{27}$
(ii) Max $= -3$, Min $= -128$
(iii) Max $= 20$, Min $= 16$ (iv) Min $= 8$
(v) Min $= -\frac{1}{e}$ (vi) Max $= \frac{1}{e}$

(10) 15, 15 (11) 10, 10 (12) 9 (13) 12.8

(14) $l = \sqrt{2}$ and $b = \frac{1}{\sqrt{2}}$

(15) Radius = Height = $a$ (16) 3, 3

(17) Side of square base = 8 cm, Height = 4 cm

(18) $x = 75, P = 4000$ (19) 6, 9

(22) $\frac{4\pi r^3}{3\sqrt{3}}$ cm$^3$

## MISCELLANEOUS EXERCISE 2

**(I)**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| A | C | B | B | D | C | D | A | D | D |

**(II)** (2) 4

(3) $14x - 13y + 12 = 0,\ 13x + 14y - 41 = 0$

(4) $\frac{2}{9\pi}$ ft/sec (5) $\left(\frac{16}{3}, 3\right), \left(-\frac{16}{3}, -3\right)$

(6) $c = 0$ (7) $c = 2$ (8) 2.025

(9) 1.03565

(10) Decreasing in $\left(0, \frac{1}{e}\right]$ and

Increasing in $\left[\frac{1}{e}, \infty\right)$

(11) Increasing in $[e, \infty)$, Decreasing in $(1, e]$

(15) $l = \frac{60}{\pi + 4}, b = \frac{30}{\pi + 4}, r = \frac{30}{\pi + 4}$

(17) Side $= \frac{l}{\pi + 4}$, Radius $= \frac{l}{2(\pi + 4)} = \frac{x}{2}$

(18) 24, 45 (21) Max $= \frac{5}{4}$, Min = 1

# 3. INDEFINITE INTEGRATION

## EXERCISE 3.1

(1) (i) $\frac{x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} + x + c$ (ii) $\frac{x^3}{3} - 2x^2 + 4x + c$

(iii) $3\tan x - 4\log x - \frac{2}{\sqrt{x}} - 7x + c$

(iv) $\frac{x^2}{4} - \frac{5x^2}{2} + 3\log x - \frac{1}{x^4} + c$

(v) $\frac{6}{5}x^2\sqrt{x} - 4\sqrt{x} - \frac{10}{\sqrt{x}} + c$

(2) (i) $\tan x - x + c$ (ii) $-2\cos x + c$

(iii) $\sec x + c$ (iv) $-\cot x - 2x + c$

(v) $-\cot x - \tan x + x + c$

(vi) $\sec x - \tan x + x + c$

(vii) $\sec x - \tan x + x + c$

(viii) $\sin x - \cos x + c$ (ix) $-\sqrt{2}\cos x + c$

(x) $-\frac{1}{14}\cos 7x - \frac{1}{2}\cos x + c$

(3) (i) $x - 2\log(x + 2) + c$

(ii) $2x + \frac{1}{2}\log(2x + 1) + c$

(iii) $\frac{5}{3}x - \frac{26}{9}\log(3x - 4) + c$

(iv) $\frac{2(x+5)^{\frac{3}{2}}}{3} - 14\sqrt{x+5} + c$

(v) $\frac{1}{12}(4x - 1)^{\frac{3}{2}} - \frac{13}{4}\sqrt{4x - 1} + c$

(vi) $-\cos 2x + c$

(vii) $\frac{2}{5}\left(\sin\frac{5x}{2} - \cos\frac{5x}{2}\right) + c$

(viii) $\frac{1}{4}(2x + \sin 2x) + c$

(ix) $-\frac{4}{9}\left[x^{\frac{3}{2}} + (x+3)^{\frac{3}{2}}\right] + c$

(x) $\frac{2}{21}\left[(7x-2)^{\frac{3}{2}} + (7x-5)^{\frac{3}{2}}\right] + c$

(4) $f(x) = \frac{x^2}{2} + \frac{3}{2x^2} + \frac{7}{2}$

## EXERCISE 3.2 (A)

**I.** 1. $\frac{(\log x)^{n+1}}{n+1} + c$ 2. $\frac{2}{5}(\sin^{-1}x)^{\frac{5}{2}} + c$

3. $\log\left(\operatorname{cosec}(x + \log x) - \cot(x + \log x)\right) + c$

4. $\frac{-1}{\sqrt{\tan(x^2)}} + c$ 5. $\frac{1}{3}(e^{3x} + 1) + c$

6. $\frac{1}{\log a} \cdot a^{x+\tan^{-1}x} + c$

7. $\frac{1}{2}\left[\log(\sin e^x)\right]^2 + c$

8. $\log(e^x - e^{-x}) + c$

9. $\frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + c$

10. $\frac{1}{48}\log\left(4x^{12}+5\right) + c$

11. $\frac{1}{10}\tan x^{10} + c$ 12. $\frac{1}{4}\log(x^4+1) + c$

13. $2\sqrt{\tan x} + c$ 14. $\tan^{-1}x + \frac{1}{x^2+1} + c$

15. $\log(3\cos^2 x + 4\sin^2 x) + c$

16. $2\tan^{-1}\sqrt{x} + c$ 17. $\log\left(10^x + x^{10}\right) + c$

18. $\frac{\sqrt{1+4x^n}}{2n} + c$

19. $\frac{4}{5}(x+2)^{\frac{5}{2}} - 2(x+2)^{\frac{3}{2}} + c$

20. $\frac{1}{7}(a^2+x^2)^{\frac{7}{2}} - \frac{4a^2}{5}(a^2+x^2)^{\frac{5}{2}} + \frac{2a^4}{3}(a^2+x^2)^{\frac{3}{2}} + c$

21. $-2\sqrt{2-3x} - \frac{2}{9}(2-3x)^{\frac{3}{2}} + c$

22. $\frac{5}{12}(2x+3)^{\frac{3}{2}} - \frac{11}{2}(2x+3)^{\frac{1}{2}} - \frac{49}{4\sqrt{2x+3}} + c$

23. $\frac{1}{3}\sin^{-1}\left(\frac{x^3}{3}\right) + c$ 25. $\frac{1}{3}\log\left(\frac{x^3-1}{x^3}\right) + c$

24. $\log(\log(\log x)) + c$

**II.** 1. $2\cdot\log\left(\sec\frac{x}{2}\right) + c$

2. $\cos a\cdot\log(\sin(x-a)) - (\sin a)\,x + c$

3. $\cos(a+b)\cdot\log(\sec(x+b)) - (\sin(a+b))\cdot x + c$

4. $\log(\tan x + 2) + c$

5. $\frac{11}{75}x + \frac{2}{25}\log(3\sin x + 4\cos x) + c$

6. $\frac{2x}{13} + \frac{3}{13}\log(2\cos x + 3\sin x) + c$

7. $5x - 3\log|2e^x - 5| + c$

8. $-5x - \log|3e^x - 4| + c$

9. $-x + \frac{7}{8}\log|4e^{2x} - 5| + c$

10. $\frac{\cos^8 x}{8} + \frac{\cos^6 x}{6} + \frac{\cos^4 x}{4} + \frac{\cos^2 x}{2} + \frac{1}{2}\log(\cos^2 x - 1) + c$

11. $\frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \log(\sec x) + c$

12. $\sin x - \sin^3 x + \frac{3}{5}\sin^5 x - \frac{1}{7}\sin^7 x + c$

13. $\frac{1}{6}\log\left[\frac{(\sec 3x)^2}{(\sec 2x)^3(\sec x)^6}\right] + c$

14. $\frac{1}{6}\cos^{11} x - \frac{1}{9}\cos^9 x + \frac{1}{13}\cos^{13} x + c$

15. $-\frac{1}{\log 3}\cdot 3^{\cos^2 x} + c$

16. $\frac{1}{20}\log\left[\frac{\sin^5 4x}{\sin^2 10x}\right] + c$

17. $\frac{1}{2}\log\left[(1+\cos^2 x) - \cos^2 x\right] + c$

## EXERCISE 3.2 (B)

**I.** 1. $\frac{1}{4\sqrt{3}}\log\left(\frac{2x-\sqrt{3}}{2x+\sqrt{3}}\right) + c$

2. $\frac{1}{30}\log\left(\frac{5+3x}{5-3x}\right) + c$

3. $\frac{1}{\sqrt{14}}\tan^{-1}\left(\frac{\sqrt{2}x}{\sqrt{7}}\right) + c$

4. $\frac{1}{\sqrt{3}}\log\left(x + \sqrt{x^2 + \frac{8}{3}}\right) + c$

5. $\frac{1}{2}\sin^{-1}\left(\frac{2x}{\sqrt{11}}\right)+c$

6. $\frac{1}{\sqrt{2}}\log\left(x+\sqrt{x^2-\frac{5}{2}}\right)+c$

7. $9\sin^{-1}\left(\frac{x}{9}\right)-\sqrt{9-x^2}+c$

8. $2\sin^{-1}\left(\frac{x}{2}\right)-\sqrt{4-x^2}+c$

9. $2\sin^{-1}\left(\frac{x}{10}\right)-\frac{1}{2}\left(\sqrt{100-x^2}\right)+c$

10. $\frac{1}{4}\log\left|\frac{x+2}{x+6}\right|+c$

11. $\frac{1}{\sqrt{5}}\log\left(\frac{\sqrt{5}-1+2x}{\sqrt{5}+1-2x}\right)+c$

12. $\frac{1}{8\sqrt{2}}\log\left(\frac{2x-5-2\sqrt{2}}{2x-5+2\sqrt{2}}\right)+c$

13. $\frac{1}{2\sqrt{19}}\log\left(\frac{3x+2+\sqrt{19}}{3x+2-\sqrt{19}}\right)+c$

14. $\frac{1}{\sqrt{3}}\log\left(x+\frac{5}{6}+\sqrt{x^2+\frac{5}{3}x+\frac{7}{3}}\right)+c$

15. $\log\left(x+4+\sqrt{x^2-8x-20}\right)+c$

16. $\frac{1}{\sqrt{2}}\log\left(x-\frac{3}{4}+\sqrt{x^2-\frac{3}{2}x+4}\right)+c$

17. $\log\left(x-\frac{1}{2}+\sqrt{x^2-x-6}\right)+c$

18. $\frac{1}{2\sqrt{7}}\tan^{-1}\left(\frac{2\tan x}{\sqrt{7}}\right)+c$

19. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\sqrt{2}\tan x\right)+c$

20. $\frac{1}{2\sqrt{3}}\log\left|\frac{\sqrt{3}+\tan x}{\sqrt{3}-\tan x}\right|+c$

**II.** 1. $\frac{2}{\sqrt{5}}\tan^{-1}\left(\frac{2\tan\frac{x}{2}+2}{\sqrt{5}}\right)+c$

2. $\frac{1}{3}\log\left[\frac{3\tan\left(\frac{x}{2}\right)-1}{3\tan\left(\frac{x}{2}\right)+1}\right]+c$

3. $\sqrt{2}\tan^{-1}\left(\frac{\tan\frac{x}{2}-1}{\sqrt{2}}\right)+c$

4. $\tan^{-1}\left[2\tan\left(\frac{x}{2}\right)+1\right]+c$

5. $\frac{1}{\sqrt{5}}\tan^{-1}\left(\sqrt{5}\tan x\right)+c$

6. $-\frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{3\tan x-2}{\sqrt{5}}\right)+c$

7. $\frac{1}{2\sqrt{11}}\log\left(\frac{\sqrt{11}-2+\tan x}{\sqrt{11}+2-\tan x}\right)+c$

8. $\frac{1}{\sqrt{2}}\log\left[\sec\left(x+\frac{\pi}{4}\right)+\tan\left(x+\frac{\pi}{4}\right)\right]+c$

9. $\frac{1}{2}\log\left[\sec\left(x+\frac{\pi}{4}\right)+\tan\left(x+\frac{\pi}{4}\right)\right]+c$

## EXERCISE 3.2 (C)

**I.** 1. $\frac{3}{2}\log(x^2+6x+5)-\frac{5}{4}\log\left(\frac{x+1}{x+5}\right)+c$

2. $\log(x^2+4x-5)-\frac{1}{2}\log\left(\frac{x-1}{x+5}\right)+c$

3. $\frac{1}{2}\log(2x^2+3x-1)+\frac{3}{2\sqrt{17}}\cdot\log\left(\frac{4x+3-\sqrt{17}}{4x+3+\sqrt{17}}\right)+c$

4. $\frac{3}{2}\sqrt{2x^2+2x+1}+\frac{5}{2\sqrt{2}}\cdot\log\left(x+\frac{1}{2}+\sqrt{x^2+x+\frac{1}{2}}\right)+c$

5. $-7\sqrt{3+2x-x^2}+10\cdot\sin^{-1}\left(\frac{x-1}{2}\right)+c$

6. $\sqrt{x^2-16x+63}+\log\left\{(x-8)+\sqrt{x^2-16x+63}\right\}+c$

7. $\sqrt{9x-x^2}+\frac{9}{2}\sin^{-1}\left(\frac{2x-9}{9}\right)+c$

8. $\frac{3}{4\sqrt{2}}\log\left(\frac{2\sqrt{2}\sin x+\sqrt{2}-2}{2\sqrt{2}\sin x+\sqrt{2}+2}\right)+c$

9. $\sqrt{e^{2x}-1}-\log\left(e^x+\sqrt{e^{2x}-1}\right)+c$

## EXERCISE 3.3

**I.** 1. $\frac{x^3}{9}(3\cdot\log x-1)+c$

2. $-\frac{x^2}{3}\cos 3x+\frac{2}{9}x\sin 3x+\frac{2}{27}\cos 3x+c$

3. $\frac{x^2}{2}\tan^{-1}x-\frac{1}{2}(x-\tan^{-1}x)+c$

4. $\frac{x^3}{3}\tan^{-1}x-\frac{x^2}{6}+\frac{1}{6}\log(1+x^2)+c$

5. $\frac{1}{4}(\tan^{-1}x)(x^4-1)-\frac{x}{12}(x^3-3x)+c$

6. $x[(\log x)^2-2(\log x)+2]+c$

7. $\frac{1}{2}\log(\sec x+\tan x)+\frac{1}{2}\sec x\cdot\tan x+c$

8. $\frac{1}{4}\left[x^2-x\cdot\sin 2x-\frac{1}{2}\cos 2x\right]+c$

9. $\frac{x^4}{4}\log x-\frac{x^4}{16}+c$

10. $\frac{e^{2x}}{13}[2\cos 3x+3\sin 3x]+c$

11. $\frac{x^2}{2}\sin^{-1}x+\frac{1}{4}x\sqrt{1-x^2}-\frac{1}{4}\sin^{-1}x+c$

12. $\frac{x^3}{3}\cos^{-1}x-\frac{1}{3}\sqrt{1-x^2}+\frac{1}{9}(1-x^2)^{\frac{3}{2}}+c$

13. $(\log x)[\log(\log x)-1]+c$

14. $-(\sin^{-1}t)\sqrt{1-t^2}+t+c$

15. $2\left[\sqrt{x}\cdot\sin\sqrt{x}+\cos\sqrt{x}\right]+c$

16. $(\cos\theta)\cdot[1-\log(\cos\theta)]+c$

17. $\frac{1}{4}\left[\frac{x}{3}\sin 3x+\frac{1}{9}\cos 3x+3x\sin x+3\cos x\right]+c$

18. $-\frac{1}{2}\cos(\log x)^2+c$

19. $-\frac{1}{2}(\log x)^2+c$

20. $\frac{x}{6}\sin 3x+\frac{1}{18}\cos x-\frac{1}{14}x\sin 7x-\frac{1}{98}\cos 7x+c$

21. $\left(3x^{\frac{2}{3}}-6\right)\sin\sqrt[3]{x}+6\sqrt[3]{x}\cos\sqrt[3]{x}+c$

**II.** 1. $\frac{e^{2x}}{13}[2\sin 3x-3\cos 3x]+c$

2. $\frac{e^{-x}}{5}[-\cos x+2\sin 2x]+c$

3. $\frac{x}{2}[\sin(\log x)-\cos(\log x)]+c$

4. $\sqrt{5}\left[\frac{x}{2}\sqrt{x^2+\frac{3}{5}}+\frac{3}{10}\log\left(x+\sqrt{x^2+\frac{3}{5}}\right)\right]+c$

5. $\frac{x^3}{6}\cdot\sqrt{a^2-x^6}+\frac{a^2}{2}\sin^{-1}\left(\frac{x^3}{a}\right)+c$

6. $\frac{x-5}{2}\sqrt{(x-3)(7-x)}+2\sin^{-1}\left(\frac{x-5}{2}\right)+c$

7. $\frac{1}{\log 2}\left\{\frac{2^x}{2}\sqrt{4^x+4}+2\log\left(2^x+\sqrt{4^x+4}\right)\right\}+c$

8. $\frac{1}{6}(2x^2+3)^{\frac{3}{2}}+\sqrt{2}\left[\frac{x}{2}\sqrt{x^2+\frac{3}{2}}+\frac{3}{4}\log\left(x+\sqrt{x^2+\frac{3}{4}}\right)\right]+c$

9. $-\frac{1}{3}(5-4x-x^2)^{\frac{3}{2}} - (x+2)\sqrt{5-4x-x^2} - 9\sin^{-1}\left(\frac{x+2}{3}\right) + c$

10. $\frac{(1+2\tan x)}{4}\sqrt{\tan^2 x+\tan x-7} - \frac{29}{8}\log\left\{\frac{1}{2}+\tan x+\sqrt{\tan^2 x+\tan x-7}\right\} + c$

11. $\left(\frac{x+1}{2}\right)\sqrt{x^2+2x+5} + 2\log\left\{x+1+\sqrt{x^2+2x+5}\right\} + c$

12. $\sqrt{2}\left\{\left(\frac{4x+3}{8}\right)\sqrt{x^2+\frac{3}{2}x+2} + \frac{23}{16\sqrt{2}}\log\left[\left(x+\frac{3}{4}\right)+\sqrt{x^2+\frac{3}{2}x+2}\right]\right\} + c$

**III.** 1. $e^x(2+\cot x)+c$ 2. $e^x\cdot\tan\frac{x}{2}+c$ 3. $e^x\cdot\frac{1}{x}+c$ 4. $e^x\cdot\left(\frac{1}{x+1}\right)+c$

5. $e^x\cdot(\log x)^2+c$ 6. $e^{5x}\cdot\log x+c$ 7. $e^{\sin^{-1}x}\cdot x+c$

8. $\frac{(1+x)^2}{2}\left(\log(1+x)-\frac{1}{2}\right)+c$ 9. $x\cdot\text{cosec}(\log x)+c$

## EXERCISE 3.4

**I.** 1. $\frac{1}{4}\log(x-1)-2\log(x+2)+\frac{11}{4}(x+3)+c$

2. $\frac{1}{6}\tan^{-1}x+\frac{1}{15\sqrt{2}}\log\left(\frac{x-\sqrt{2}}{x+\sqrt{2}}\right)-\frac{\sqrt{3}}{10}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right)+c$

3. $\frac{51}{41}\log(2x+9)+\frac{31}{41}\log(3x-7)+c$ 4. $-\frac{8}{5}\log(x+4)-\frac{2}{5}\log(x-1)+c$

5. $x-\log(x+3)+\log(x-2)+c$ 6. $x^2+3x+\frac{5}{3}\log(3x+1)+\log(x-1)+c$

7. $\frac{1}{2}\log\left|\frac{2x+1}{2x-1}\right|+3\log(x+3)+c$ 8. $\frac{1}{5}\log\left(\frac{x^5}{x^5+1}\right)+c$ 9. $\frac{11}{\sqrt{5}}\tan^{-1}\left(\frac{x}{2}\right)-\frac{9}{2}\tan^{-1}\left(\frac{x}{2}\right)+c$

10. $2\log\left(\frac{x+1}{x-1}\right)+\frac{5}{2\sqrt{2}}\log\left(\frac{x+\sqrt{2}}{x-\sqrt{2}}\right)+c$ 11. $\log\left(\frac{2+x^2}{3+x^2}\right)+c$

12. $\frac{1}{5\cdot\log 2}\log\left(\frac{2^x-4}{2^x+1}\right)+c$ 13. $\frac{5}{2}\left(\frac{1}{x+1}\right)+\frac{11}{4}\log\left(\frac{x+1}{x+3}\right)+c$

14. $6\cdot\log x-\log(x+1)-\frac{9}{x+1}+c$ 15. $\frac{1}{8}\log\left(\frac{x^6(x^3+3)}{(3x^3+1)^3}\right)+c$

16. $\frac{1}{3}\log(x-1)-\frac{1}{6}\log(x^2+x+1)-\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+c$

17. $3 \cdot \log(\sin x - 2) - \dfrac{4}{\sin x - 2} + c$

18. $\dfrac{1}{2}\log(\cos x + 1) + \dfrac{1}{6}\log(\cos x - 1) - \dfrac{2}{3}\log(2\cos x + 1) + c$

19. $\dfrac{1}{8}\log\left(\dfrac{\cos x - 1}{\cos x + 1}\right) + \dfrac{1}{4 \cdot (\cos x + 1)} + c$

20. $\dfrac{1}{6}\log\left[\dfrac{(1 + 2\sin x)^4}{(1 - \sin x)(1 + \sin x)^3}\right] + c$

21. $\dfrac{1}{10}\log(1 - \cos x) - \dfrac{1}{2}\log(1 + \cos x) + \dfrac{2}{5}\log(3 + 2\cos x) + c$

22. $\dfrac{1}{2}\log\left[\dfrac{e^x + 1}{(e^{2x} + 9)^{\frac{1}{2}}}\right] + \dfrac{1}{6}\tan^{-1}\left(\dfrac{e^x}{3}\right) + c$

23. $\dfrac{5}{26}\log\left[\dfrac{(3\log x + 2)^2}{\sqrt{(\log x)^2 + 1}}\right] + \dfrac{11}{26}\tan^{-1}(\log x) + c$

## MISCELLANEOUS EXERCISE 3

**(I)**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| B | A | B | A | D | B | A | A | C | B |

| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|
| A | A | D | C | A | D | A | D | C | A |

**(II)** (1) $\dfrac{2}{7}x^{\frac{7}{2}} - \dfrac{8}{5}x^{\frac{5}{2}} - \dfrac{8}{3}x^{\frac{3}{2}} + c$

(2) $\dfrac{x^7}{7} - \dfrac{x^6}{6} + \dfrac{x^5}{5} - \dfrac{x^4}{4} + \dfrac{x^3}{3} - \dfrac{x^2}{2} + x - \log(x + 1) + c$

(3) $\dfrac{1}{15}(6x + 5)^{\frac{5}{2}} + c$

(4) $\dfrac{t^2}{2} - 2t + 3 \cdot \log(t + 1) + \dfrac{1}{t + 1} + c$

(5) $3\tan x - 2\sec x + c$

(6) $\tan\theta - \cot\theta - 3\theta + c$

(7) $\dfrac{1}{48}(2\sin 6x + 3\sin 4x + 6\sin 2x + 12x) + c$

(8) $\dfrac{1}{2}\sin 2x - \dfrac{1}{3}\sin 3x + c$

(9) $\dfrac{\pi}{4}x - \dfrac{1}{4}x^2 + c$

**(III)** (1) $\dfrac{1}{4}(1 + \log x)^4 + c$

(2) $(\tan^{-1}x)\,x - \dfrac{1}{2}\log(1 + x^2) - (1 - x)\tan^{-1}(1 - x) + \dfrac{1}{2}\log(x^2 - 2x + 2) + c$

(3) $-\cot(\log x) + c$

(4) $\dfrac{2}{3}\sec x^{\frac{3}{2}} + c$

(5) $x \log(1+\cos x)+c$

(6) $\frac{1}{3}\sin^{-1}(x^3)+c$

(7) $\frac{1}{4}\log(3-2\cot x)+c$

(8) $x\cdot\left(\log(\log x)-\frac{1}{\log x}\right)+c$

(9) $\frac{2}{\sqrt{13}}\tan^{-1}\left(\frac{2\tan\left(\frac{x}{2}\right)-3}{\sqrt{13}}\right)+c$

(10) $\frac{1}{4}\left(2\sec^{-1}x+\frac{2\sqrt{x^2-1}}{x^2}\right)+c$

(11) $-\frac{3}{2}\sqrt{-2x^2+x+3}+\frac{7}{4\sqrt{2}}\sin^{-1}\left(\frac{2x-1}{\sqrt{7}}\right)+c$

(12) $x\cdot\log(x^2+1)-2\left[x-\tan^{-1}x\right]+c$

(13) $\frac{1}{4}e^{2x}\cdot\left[\sin 2x-\cos 2x\right]+c$

(14) $\frac{1}{18}\log(3x-1)+\frac{1}{2}\log(x-1)-\frac{4}{9}\log(3x-2)+c$

(15) $\frac{1}{6}\log\left\{\frac{(\cos x-1)(\cos x+1)^3}{(2\cos x+1)^4}\right\}+c$

(16) $\left(\frac{\tan x-1}{2}\right)\sqrt{7+2\tan x-\tan^2 x}+4\sin^{-1}\left(\frac{\tan x-1}{2\sqrt{2}}\right)+c$

(17) $\frac{1}{4}\log\left\{\frac{(x-1)^3(x+3)}{(x+1)^4}\right\}+c$

(18) $\frac{1}{5}\log\left(\frac{x^5}{x^5+1}\right)+c$

(19) $2\sqrt{\tan x}+c$

(20) $\frac{1}{3\cot^3 x}+\frac{2}{\cot x}-\cot x+c$

# 4. DEFINITE INTEGRATION

## EXERCISE 4.1

I. (1) 4 (2) $\frac{64}{3}$ (3) $e^2-1$ (4) 6 (5) 20

## EXERCISE 4.2

I. (1) $\frac{64}{3}$ (2) $\log\left(\frac{25}{24}\right)$ (3) $-\left(1+\frac{\pi}{4}\right)$ (4) 2

(5) $\frac{1}{18}\left[13\sqrt{13}+7\sqrt{7}-3\sqrt{3}-27\right]$ (6) $1-\frac{3\pi}{4}$ (7) $\frac{4}{7\sqrt{2}}$ (8) 1 (9) $\frac{3\pi}{16}$

(10) $\frac{1}{3}\left(\tan^{-1}\frac{4}{3}+\tan^{-1}\frac{2}{3}\right)$ (11) $\pi$ (12) $\frac{\pi}{6}$ (13) 1 (14) $\frac{\pi}{4}-\frac{1}{2}$ (15) 1

II. (1) $\frac{\pi}{4} - \frac{1}{2}\log 2$ (2) $\frac{1}{2}\log 2$ (3) $\frac{\pi}{4}$

(4) 0 (5) $\frac{2}{3}\tan^{-1}\left(\frac{1}{3}\right)$ (6) $\frac{1}{4}\log\left(\frac{2\sqrt{2}+1}{2\sqrt{2}-1}\right)$

(7) $\log\left(\frac{4}{3}\right)$ (8) $\frac{1}{ab}\left[\tan^{-1}\left(\frac{ae}{b}\right) - \tan^{-1}\left(\frac{a}{be}\right)\right]$

(9) $\frac{\pi}{4}$ (10) $\frac{4}{3}$ (11) $\frac{\pi}{2} - 1$

(12) $\frac{8}{3}$ (13) $\frac{\pi}{2} - 1$

(14) $e^{\frac{\pi}{4}}\left[\frac{\pi}{4}+1\right] - \left[\frac{\pi}{2}+1\right]$ (15) $\sin(\log 3)$

III. (1) $\frac{\pi}{4}$ (2) 0 (3) 0 (4) 0 (5) $\frac{16}{77}(3)^{\frac{7}{2}}$

(6) 0 (7) 0 (8) $\frac{\pi^2}{6\sqrt{3}}$ (9) 0 (10) 0

(11) $4\log\left(\frac{1+\sqrt{5}}{2}\right)$ (12) 0 (13) $\frac{16}{105}$ (14) $\frac{\pi}{3}$ (15) $\frac{\pi}{2}\log\left(\frac{1}{2}\right)$

## MISCELLANEOUS EXERCISE 4

**(I)**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| A | A | C | C | D | C | A | D | B | A |

**(II)** (1) $\frac{1}{10}(3 - \log 3)$ (2) $2 - \sqrt{2}$ (3) $6 - 4\log 2$ (4) $\frac{1}{8}$ (5) $\frac{1}{21}$

(6) $\pi - 2$ (7) $\frac{1}{3}\log 2$ (8) $\frac{\pi}{5}$ (9) 0 (10) $\frac{\pi}{2}$

**(III)** (1) $\frac{\pi^2}{16}$ (2) $\frac{2}{\sqrt{35}}\tan^{-1}\sqrt{\frac{7}{5}}$ (3) $\frac{1}{\sqrt{5}\,a}\log\left(\frac{7+3\sqrt{5}}{2}\right)$ (4) $\frac{\pi}{20}$

(5) $\frac{\pi}{2} - \log 2$ (6) $\frac{1}{2}\left(\frac{\pi}{4} - \log\sqrt{2}\right)$ (7) $-\frac{\pi}{2}\log 2$ (8) $\frac{\pi^3}{6}$

(9) $\log\left(\frac{5+3\sqrt{3}}{1+\sqrt{3}}\right)$ (10) $\frac{17}{2}$

**(IV)** (1) $\frac{1}{2}$ when $a = 0$ ; $\frac{9}{2}$ when $a = 4$ (2) $k = \frac{1}{2}$

# 5. APPLICATION OF DEFINITE INTEGRAL

## EXERCISE 5.1

(1) (i) 25 (ii) 16 (iii) 20

(iv) 1 (v) $2 \log 4$ (vi) $\frac{32}{3}$

(vii) $\frac{128}{3}$ sq. units

(2) (i) $\frac{128}{3}$ (ii) $\frac{16}{3}$

(3) (i) $\frac{1}{12}$ (ii) $\frac{8}{3}$ (iii) $\frac{32}{3}$

(iv) $8\frac{a^2}{3}$ (v) $\frac{1}{6}$

## MISCELLANEOUS EXERCISE 5

**(I)**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| A | A | C | B | A | D | B | D | A | B |

| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|
| A | D | B | B | C | C | A | D | A | C |

**(II)**

1. (i) 10 (ii) 2 (iii) $\frac{1}{2}$

2. $9\pi$ 3. $20\pi$

4. (i) $\frac{16}{3}$ (ii) $\frac{8}{3}$ (iii) $\frac{1}{3}$

5. $\frac{\pi}{3}$ 6. $\frac{1}{6}$ 7. $\frac{\pi}{4} - \frac{1}{2}$

8. $\frac{56}{3}$ 9. $36\frac{3}{4}$ 10. $\frac{7}{3}$

# 6. DIFFERENTIAL EQUATIONS

## EXERCISE 6.1

(1) (i) 2, 1 (ii) 2, 3 (iii) 1, 2

(iv) 3, 1 (v) 2, 1 (vi) 3, 2

(vii) 2, not definded (viii) 2, 2

(ix) 3, 3 (x) 2, 1

## EXERCISE 6.2

(1) (i) $2x^3 + 3xy^2\frac{dy}{dx} - y^3 = 0$

(ii) $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

(iii) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ (iv) $8\left(\frac{dy}{dx}\right)^3 - 27y = 0$

(v) $\frac{d^2y}{dx^2} - 25y = 0$ (vi) $2\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$

(vii) $(x^2+xy)\frac{dy}{dx} + y = 0$ (viii) $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 10y = 0$

(ix) $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - 2y\frac{dy}{dx} = 0$

(x) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$

(2) $\frac{d^2y}{dx^2} = 0$ (3) $2a\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$

(4) $x + 4y\frac{dy}{dx} = 0$ (5) $3\frac{dy}{dx} + 2 = 0$

(6) $81\left(\frac{d^2y}{dx^2}\right)^2 = \left[\left(\frac{dy}{dx}\right)^2 + 1\right]^3$

(7) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

## EXERCISE 6.3

(2) (i) $\tan^{-1} y = \tan^{-1} x + c$

(ii) $2e^{-3y} + 3e^{2x} = c$ (iii) $x = cy$

(iv) $\tan x \cdot \tan y = c$ (v) $\sin y \cdot \cos x = c$

(vi) $y = -kx + c$

(vii) $2(x^2 + y^2) + 2(x \sin 2x + y \sin 2y) + \cos 2y + \cos 2x + c = 0$

(viii) $2y^2 \tan^{-1} x + 1 = cy^2$

(ix) $4e^x + 3e^{-2y} = c$

(x) $3e^x + 3e^{-y} + x^3 = c$

(3) (i) $(1 + e^x)^3 \tan y = 0$

(ii) $(1 + x^2)(1 - y^2) = 5$

(iii) $y = ex \log x$ (iv) $(\sin x)(e^y + 1) = \sqrt{2}$

(v) $2(2 + e^y) = 3(x + 1)$

(vi) $\cos\left(\frac{y-2}{x}\right) = a$

(4) (i) $\tan\left(\frac{x+y}{2}\right) = x + c$

(ii) $c + 2y = a \log\left(\frac{x-y-a}{x-y+a}\right)$

(iii) $\sin(x^2 + y^2) + 2x = c$

(iv) $x = \tan(x - 2y) + c$

(v) $(2x - y) - \log(x - y + 2) + 1 = 0$

## EXERCISE 6.4

(1) $\cos\left(\frac{y}{x}\right)dy = \log(x) + c$

(2) $x^2 - y^2 = cx$ (3) $x + 2ye^{\frac{x}{y}} = c$

(4) $xy^2 = c^2(x + 2y)$ (5) $x^2 + y^2 = cx$

(6) $y = c(x + y)^3 + x$

(7) $x\left[1 - \cos\left(\frac{y}{x}\right)\right] = \sin\left(\frac{y}{x}\right)$

(8) $x + ye^{\frac{x}{y}} = c$ (9) $\log(y) + \frac{y}{x} = c$

(10) $x^2y = 4$ (11) $x^2 + y^2 = x^4$

(12) $\tan^{-1}\left(\frac{y}{x}\right) = \log(x) + c$

(13) $(3x + y)^3 (x + y)^2 = c$

(14) $c = \log(x) + \frac{x}{x+y}$ (15) $x^2 - y^2 = cx$

## EXERCISE 6.5

1. (i) $\frac{x^5}{5} - \frac{3x^2}{2} - xy = c$

(ii) $ye^{\tan x} = e^{\tan x}(\tan x - 1) + c$

(iii) $x = y(c + y^2)$

(iv) $y(\sec x + \tan x) = \sec x + \tan x - x + c$

(v) $x^2y = \frac{x^4 \log x}{4} - \frac{x^4}{16} + c$

(vi) $x + y + 1 = ce^y$

(vii) $2y = (x + a)^5 + 2c(x + a)^3$

(viii) $r \sin^2 \theta + \frac{\sin^4 \theta}{2} = c$

(ix) $\frac{y^3}{3} = xy + c$

(x) $y = \sqrt{1 - x^2} + c(1 - x^2)$

(xi) $y = \frac{1}{2}e^{\tan^{-1} x} + c\,e^{-\tan^{-1} x}$

2. $3(x+3y) = 2(1-e^{3x})$

3. $4x^2 + 9y^2 = 36$

4. $y = 4 - x - 2e^x$

5. $1 + y = 2e^{\frac{x^2}{2}}$

## EXERCISE 6.6

1. 8 times of original.
2. 95·4 years
3. 36·36°c
4. 5656
5. $\frac{\log 3}{k}$
6. $\frac{27}{5}$ gms
7. $(3000)\left(\frac{4}{9}\right)^{\frac{t}{40}}$
8. 1 hour
10. $r = 3 - t$
11. 27,182
12. $\left(10 - \frac{p}{10}\right)^2$ %

## MISCELLANEOUS EXERCISE 6

**(I)**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D | A | C | B | A | D | C | B | C | D | B | A | B | B | B |

**(II)** (1) (i) 2, 1 (ii) 3, 10 (iii) 2 , 3 (iv) 1.4 (v) 4, not defined

(3) (i) $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - 2y\frac{dy}{dx} = 0$ (ii) $\frac{d^2y}{dx^2} + y = 0$ (iii) $(y-a)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

(iv) $2x^2y\frac{d^2y}{dx^2} + 2x^2\left(\frac{dy}{dx}\right)^2 + y = 0$ (v) $\frac{d^2y}{dx^2} - 9y = 0$

(4) (i) $2xy\frac{dy}{dx} + x^2 - y^2 = 0$ (ii) $2b\frac{d^2y}{dx^2} - 1 = 0$ (iii) $x + 4y\frac{dy}{dx} = 0$ (iv) $2\frac{dy}{dx} - 3 = 0$

(5) (i) $2e^{-3y} + 3e^{2x} + 6c = 0$ (ii) $\log(y) = \frac{x^3}{3} + x + c$ (iii) $y = \frac{x}{2}\log(x^2) + 2 + cx$

(iv) $y = 1 + x\log x + cx$ (v) $y = x^2 + c\cdot\operatorname{cosec} x$ (vi) $x\log y = (\log y)^2 + c$

(vii) $4xe^{2y} + 5e^{-y} = c$

(6) (i) $ex\log x - y = 0$ (ii) $x = 2y^2$ (iii) $y\operatorname{cosec}^2 x + 2 = 4\sin 2x$

(iv) $\log\sqrt{x^2+y^2} + \tan^{-1}\left(\frac{y}{x}\right) = \frac{\pi}{4}$ (v) $x + 2ye^{\frac{x}{y}} = 2$

(8) $x^2 + y^2 = 4x + 5$ (9) $r = (63t + 27)^{\frac{1}{3}}$ (10) $\frac{20}{9}$ years

# 7. PROBABILITY DISTRIBUTIONS

## EXERCISE 7.1

1. $\{-6, -4, -2, 0, 2, 4, 6\}$

2. $\{0, 1, 2\}$

3. (i) p.m.f. (ii) Not p.m.f
   (iii) p.m.f (iv) Not p.m.f
   (v) Not p.m.f. (iv) p.m.f

4. (i)

| $X$ | 0 | 1 | 2 |
|---|---|---|---|
| $P(X)$ | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{1}{4}$ |

(ii)

| $X$ | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $P(X)$ | $\frac{1}{8}$ | $\frac{3}{8}$ | $\frac{3}{8}$ | $\frac{1}{8}$ |

(iii)

| $X$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X)$ | $\frac{1}{16}$ | $\frac{1}{4}$ | $\frac{3}{8}$ | $\frac{1}{4}$ | $\frac{1}{16}$ |

5.

| $X$ | 0 | 1 | 2 |
|---|---|---|---|
| $P(X)$ | $\frac{2}{3}$ | $\frac{2}{9}$ | $\frac{1}{9}$ |

6.

| $X$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X)$ | $\left(\frac{4}{5}\right)^4$ | $\left(\frac{4}{5}\right)^3 \frac{1}{5}$ | $\left(\frac{4}{5}\right)^2 \left(\frac{1}{5}\right)^2$ | $\frac{4}{5}\left(\frac{1}{5}\right)^3$ | $\left(\frac{1}{5}\right)^4$ |

7.

| $X$ | 0 | 1 | 2 |
|---|---|---|---|
| $P(X)$ | $\frac{9}{16}$ | $\frac{3}{8}$ | $\frac{1}{16}$ |

8. (i) $\frac{1}{10}$ (ii) $\frac{3}{10}$ (iii) $\frac{1}{5}$

9. $-0{\cdot}05, 2{\cdot}2475$ 10. $\frac{7}{3}, \frac{524}{54}$ 11. $1{\cdot}5$

12. $\frac{1}{3}$ 13. $4{\cdot}67$ 14. $2{\cdot}41$

15. $17{\cdot}53, 4{\cdot}9, 2{\cdot}21$ 16. $0{\cdot}7, 0{\cdot}21$

## EXERCISE 7.2

1. (i) p.d.f. (ii) Not a p.d.f
   (iii) Not a p.d.f

2. (a) $\frac{2{\cdot}25}{16}$, (b) $\frac{3}{16}$, (c) $\frac{3}{4}$

3. (i) p.d.f. (ii) $\frac{1}{9}$ (iii) $\frac{1}{9}$

4. (i) $\frac{1}{2}, \frac{35}{64}$ (ii) $6, \frac{11}{32}, \frac{1}{2}$

5. (i) $\frac{1}{4}$ (ii) $\frac{1}{2}$ (iii) $\frac{7}{16}$

6. (i) $\frac{2}{5}$ (ii) $\frac{1}{5}$

7. (i) $\frac{1}{2}$ (ii) $\frac{11}{16}$ (iii) 0·6328

8. (i) $\frac{x^2}{16}$ (ii) $\frac{1}{64}$, 0·18, 1

9. $\frac{2}{9}, 0, \frac{8}{9}, \frac{7}{9}$

10. $\frac{1}{\log 3}, \frac{4}{\log 3}, \frac{4(\log 3 - 1)}{(\log 3)^2}$

## MISCELLANEOUS EXERCISE 7

**(I)**

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| B | C | A | B | C | B | B | A | D | B |

**(II) Solve the following :**

(1) (i) Discrete $\{1, 2, 3, ..., 100000\}$ (ii) Continuous. (iii) Continuous.

(iv) Discrete $\{0, 1, 2, 3, 4, 5\}$ (v) Continuous

(2) (i) $\frac{1}{21}$ (ii) $\frac{10}{21}, \frac{1}{7}, \frac{6}{7}$ (3) (i) 0·5 (ii) 0·7 (iii) 0·55 (iv) 0·45

(5) $\frac{1}{4}$

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $P(X)$ | $\frac{1}{20}$ | $\frac{3}{20}$ | $\frac{1}{4}$ | $\frac{1}{2}$ | $\frac{1}{20}$ |
| $F(x)$ | $\frac{1}{20}$ | $\frac{1}{5}$ | $\frac{9}{20}$ | $\frac{19}{20}$ | 1 |

(6)

| $X$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X)$ | $\frac{1}{16}$ | $\frac{1}{4}$ | $\frac{3}{8}$ | $\frac{1}{4}$ | $\frac{1}{16}$ |

$\frac{{}^4C_x}{2^4}$

(7) (i)

| $X$ | 0 | 1 | 2 |
|---|---|---|---|
| $P(X)$ | $\frac{4}{9}$ | $\frac{4}{9}$ | $\frac{1}{9}$ |

(ii)

| $X$ | 0 | 1 | 2 |
|---|---|---|---|
| $P(X)$ | $\frac{25}{36}$ | $\frac{10}{36}$ | $\frac{1}{36}$ |

(8) (i) $\frac{1}{10}$ (ii) $\frac{17}{100}$ (iii) $\frac{3}{10}$

(9)

| $X$ | –3 | –2 | –1 | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|
| $F(X)$ | 0·1 | 0·3 | 0·5 | 0·65 | 0·75 | 0·85 | 0·9 | 1 |
| $P(x)$ | 0·1 | 0·2 | 0·2 | 0·15 | 0·10 | 0·10 | 0·05 | 0·10 |

(i) 0·55 (ii) 0·25

(10) (i) $\frac{11}{5}, \frac{14}{25}, \frac{\sqrt{14}}{5}$ (ii) $\frac{1}{5}, \frac{14}{25}, \frac{\sqrt{14}}{5}$ (iii) $\frac{n+1}{2}, \frac{n^2-1}{12}, \sqrt{\frac{n^2-1}{12}}$ (iv) $\frac{5}{2}, \frac{5}{4}, \frac{\sqrt{5}}{2}$

(11) ₹ 5·5, 8·25 (12) 0, 1 (13) (i) $\frac{1}{2}$ (ii) $\frac{11}{16}$ (iii) $\frac{81}{128}$

(15) $k = \frac{1}{\theta}, \frac{1}{e}$ (16) $k = \frac{1}{4}, F(x) = \frac{\sqrt{x}}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}$

# 8. BINOMIAL DISTRIBUTION

## EXERCISE 8.1

1. (i) $\frac{3}{2^5}$ (ii) $\frac{7}{2^6}$ (iii) $\frac{63}{64}$

2. $\frac{25}{216}$ 3. $29\left(\frac{19^9}{20^{10}}\right)$

4. (i) $\frac{1}{1024}$ (ii) $\frac{45}{1024}$

5. (i) $(0{\cdot}95)^5$ (ii) $(1{\cdot}2)(0{\cdot}95)^4$
(iii) $1-(1{\cdot}2)(0{\cdot}95)^4$ (iv) $1-(0{\cdot}95)^5$

6. $\left(\frac{9}{10}\right)^4$ 7. $\frac{11}{243}$

8. (i) $1-\left(\frac{99}{100}\right)^{50}$ (ii) $50\left(\frac{99^{49}}{100^{50}}\right)$
(iii) $1-149\left(\frac{99^{49}}{100^{50}}\right)$

9. (i) $\frac{1}{20^3}$ (ii) $3\left(\frac{19}{20^3}\right)$ (iii) $3\left(\frac{19^2}{20^3}\right)$ (iv) $\left(\frac{19}{20}\right)^3$

10. $\frac{7}{3}\left(\frac{5}{6}\right)^5$ 11. $22\left(\frac{9^3}{10^{11}}\right)$

12. (i) 4, 2·4 (ii) 10, 2·4 (iii) $\frac{2}{5}; \sqrt{6}$ (iv) $\frac{8}{5}$

## MISCELLANEOUS EXERCISE 8

(I)

| 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|
| B | D | D | C | B | C | B |

**(II) Solve the following :**

(1) (i) $2 \times (0{\cdot}8)^9$ (ii) $1 - (0{\cdot}8)^{10}$

(iii) $1 - (8{\cdot}2)(0{\cdot}2)^9$

(2) (i) $p = \frac{1}{2}$, $Var(X) = 2{\cdot}5$

(ii) $n = 10, p = \frac{1}{2}$

(3) (i) $\frac{63}{256}$ (ii) $\frac{105}{512}$

(4) $45\left(\frac{2^{26}}{2^{10}}\right)$

(5) (i) $0{\cdot}65 \times (0{\cdot}95)^{16}$

(ii) $(2{\cdot}0325) \times (0{\cdot}95)^{14}$

(iii) $1 - (1{\cdot}6) \times (0{\cdot}95)^{16}$

(6) $0{\cdot}2114$ (7) $1{\cdot}4 \times (0{\cdot}9)^4$

(8) $6{\cdot}97 \times (0{\cdot}97)^{19}$

(9) $0{\cdot}3456$ (10) $\frac{30{\cdot}44}{5^8}$

(11) (i) $(0{\cdot}998)^8$ (ii) $1{\cdot}014 \times (0{\cdot}998)^7$

(iii) $1 - 1{\cdot}014 \times (0{\cdot}998)^7$

(12) $775{\cdot}44 \times (0{\cdot}003)^{38}$

(13) (i) $0{\cdot}9^{10}$ (ii) $0{\cdot}9^9$

(iii) $0{\cdot}45 \times (0{\cdot}9)^8$

(iv) $1 - 2{\cdot}16 \times (0{\cdot}9)^8$

(14) (i) $\frac{1}{5^4}, \frac{16}{5^4}, \frac{96}{5^4}, \frac{256}{5^4}, \frac{256}{5^4}$

(ii) (a) $\frac{608}{5^4}$ (b) $1 - \frac{33}{5^8}$

(15) (i) $35 \times 8 \times \frac{81}{5^7}$ (ii) $1 - \frac{12393}{5^7}$

(16) (i) $\frac{\log 0{\cdot}5}{\log 0{\cdot}99}$

(17) $\frac{1}{5}$

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